An artistic rendering of the Pioneer 10 spacecraft in space. The spacecraft is a complex of various instruments and antennas, including a large parabolic dish antenna and a long boom with a smaller antenna at the end. It is positioned in the foreground, with the curved horizon of a large, reddish-brown planet (likely Jupiter) in the background. The background is a dark, star-filled space.

Pioneer spacecraft thermal analysis

Thermal model principles

- Heat flows from sources to space, and obeys conservation of energy
1. Enumerate heat sources
 2. Estimate (using telemetry) their power
 3. Develop model that translates power into acceleration
 4. Perform numerical calculations

Power vs. temperature

- Telemetry includes temperature sensors
- BUT: these are spot measurements, not reliable average/ambient temperatures
- Thrust is a function of power ($a = P/mc$), only indirectly a function of temperature
- Temperature-based calculations face uncertainties
 - Errors magnified by T^4 relationship
 - Emissivity may not be known (or known only approximately)
- Power is known from telemetry and conservation of energy
- At the very least, if temperatures are used, overall power output should be calculated and used for calibration

Heat sources

- Electrical heat
- RTGs
- RHUs
- Antenna emission
- Thrusters

Electrical heat

- Most electrical instrumentation inside spacecraft body
- Devices mounted outside tend to radiate sideways → No significant spin-axis force
- Spacecraft body covered with MLI; outside temperature differences likely small
- Estimate heat through MLI and through louver system

RTGs

- RTG total power is known (fuel inventory established prior to launch)
- RTG electrical power is known from telemetry

$$P_{\text{TOTAL}} - P_{\text{ELECTRICAL}} = P_{\text{WASTE HEAT}}$$

- RTG radiation pattern is function of shape and fin structure, but fore-aft symmetrical
- Force is due to radiation intercepted and reflected by HGA

RHUs

- ^{238}Pu fuel capsules as heating elements
- Total power is known (1 W nominal at launch, 11 RHUs)
- Geometry uncertain

Antenna emission

- Transmitter power is present in telemetry
- Antenna efficiency can be estimated
- Transmitter power must be subtracted from electrical power

Thrusters

- When thrusters are fired, thruster assemblies heat up several 100 degrees
- BUT: event is transient and dwarfed by the uncertainties in the thruster event itself
- Conclusion: effects of thruster heat absorbed into maneuver uncertainty; no need to model thruster heating

Heat from telemetry

- $P_{\text{RTG(heat)}} = P_{\text{RTG(total)}} - P_E$
total heat is calculated, electrical power
obtained from telemetry
- $P_{E(\text{heat})} = \sum P_{i(\text{body})}$
total power consumption in spacecraft
body can be computed from telemetry
- P_{RHU} (calculated)
- P_{TWT} (from telemetry)

How to estimate thermal thrust?

- Model is simple (spinning spacecraft)
- Model is linear (unchanging configuration)
- Three substantially independent estimation methods possible:
 - Analytical
 - Numerical
 - Parameter fitting

Significance of spin

- Thermal forces are slowly changing. Rate of change much smaller than angular velocity: $(dF/dt)/F \ll \omega/\pi$
- Force components perpendicular to spin axis average to zero to first order
- Hence only spin axis component of thermal forces needs to be computed

Why is the model linear?

- No significant trapped heat relative to the rate of change in temperatures (no latency)
- No significant variability in the emission/absorption spectrum of materials at spacecraft temperatures
- Physical configuration of spacecraft and mass constant during deep space cruise

Linear dependence

- Linear dependence means force can be expressed as a function of the power of heat sources and constant coefficients:

$$F = (1/c)\Sigma q_i P_i$$

- **Goal: determine the q_i**

Analytical model

- Heat transfer is described by a double surface integral:

$$P_{1 \rightarrow 2} = \iint P_1 \cos \chi_1 \cos \chi_2 / \pi r^2 dA_1 dA_2$$

- This can be simplified and evaluated if
 - We approximate the HGA with a simplified geometry
 - We approximate the RTGs as cylindrically symmetric anisotropic point sources
- Errors introduced by these simplifications can be estimated

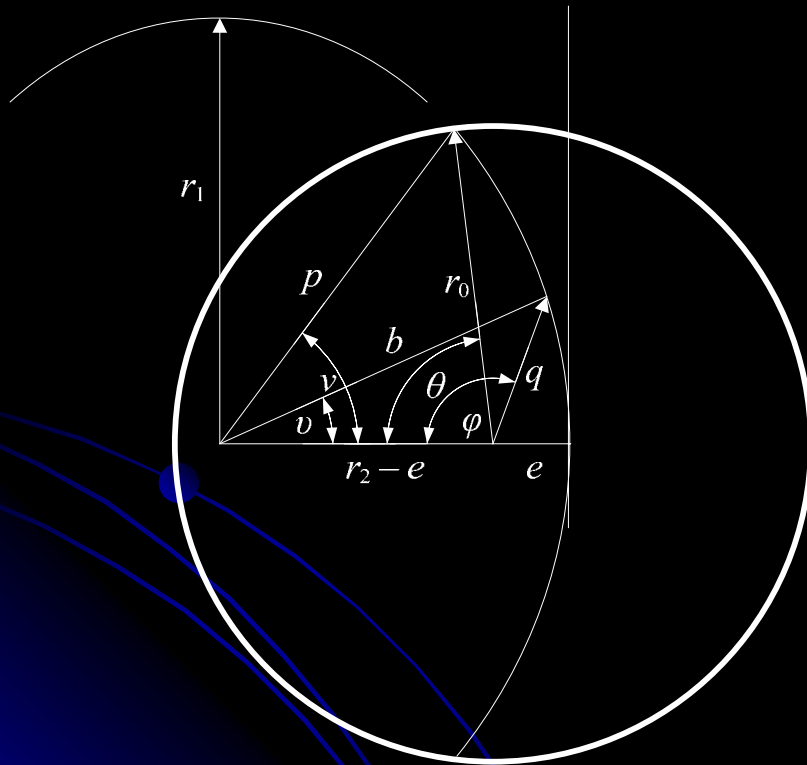
Analytical model (continued)

- Separate integral expressions can be derived for the force due to:
 - Incident radiation
 - Absorbed and re-emitted radiation
 - Diffusely reflected radiation
 - Specularly reflected radiation
- Analytic expression for antenna portion illuminated by RTG can be derived

Analytical model (continued)

- Effect due to the shadow of the spacecraft body can be approximately estimated
- Heat emitted by the spacecraft body can be estimated from surface area covered by MLI vs. louvers
- Results of the above can be evaluated using a computer algebra system (CAS)

Geometry and CAS

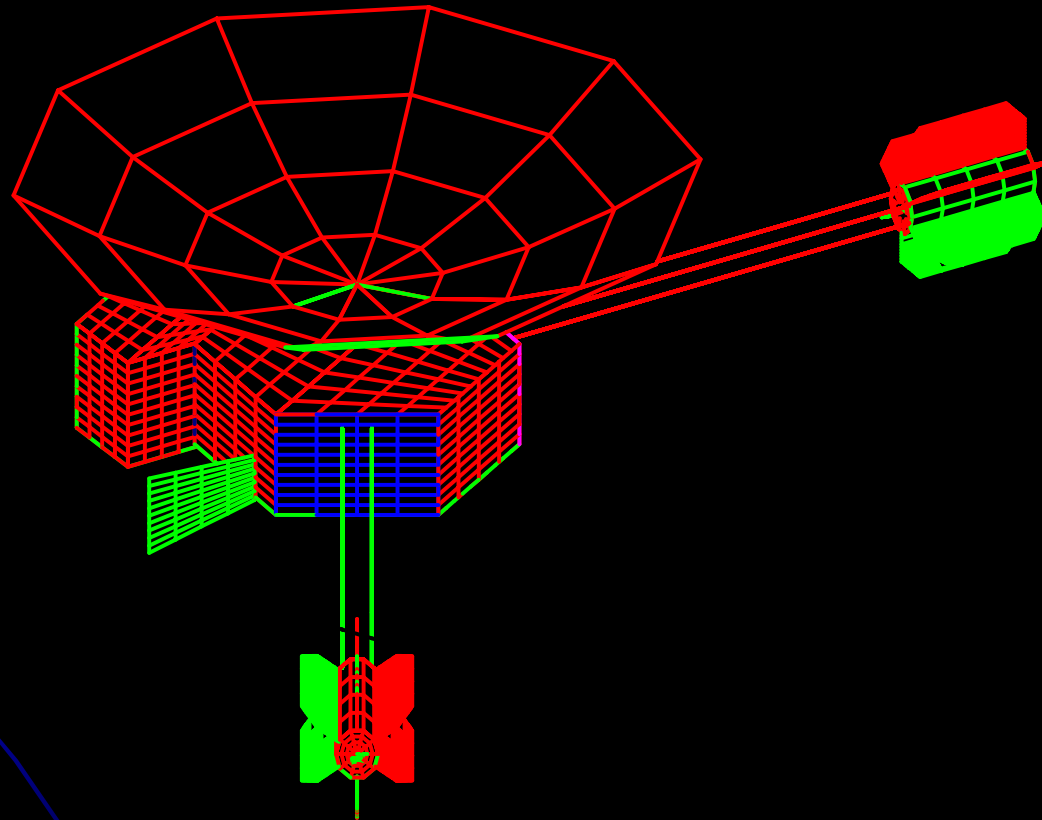


```
with(student);
al pha=0.04;
r0:=54;
h:=17;
d:=11;
R:=eval f((r0^2+h^2)/2/h);
ls[1]:=-97.3;
ls[2]:=-102.85;
ls[3]:=-114.95;
zetas[1]:=-0.097/4.*cos('del ta');
zetas[2]:=-0.596/4.*sin('del ta');
zetas[3]:=-0.596/4.*sin('del ta');
for k from 1 to 3 do
l:=ls[k]; zeta:=zetas[k];
r1:=R*sqrt((R-d)^2+1^2)/sqrt((R-d)^2+1^2);
e:=r1*sqrt((R+d)^2+1^2)/(R+d)-1/R/(R+d);
r2:=r1*(R-d)/sqrt((R+d)^2+1^2);
m:=sqrt((R+d)^2+1^2);
s:=R-sqrt(R^2-r^2);
n:=sqrt(1^2+r^2-2*1*r*cos(al pha));
u:=sqrt(n^2+(d+s)^2);
coschi:=-(u^2+R^2-m^2)/(2*R*u);
E01:=-p*(r1^2+(1-S)+r2^2*S)=r1^2*r2^2;
E02:=-p*(r2-e)^2+r0^2-2*(r2-e)*r0*c;
E03:=-S/r0^2-(1-C^2)/p;
use Real Domain 1 in res1:=sol ve({E01, E02, E03, -1<C, C<=1}, {P, C, S}) end;
theta:=arccos(eval ([C], res1)[1]);
E04:=-b^2*(r1^2*cos(mu)^2+r2^2*sin(mu)^2)=r1^2*r2^2;
E05:=-b^2*(r2-e)^2+q^2-2*(r2-e)*q*cos(al pha);
E06:=sin(mu)/q=sin(al pha)/b;
use Real Domain 1 in res1:=sol ve({E04, E05, E06}, {b, mu, q}) end;
res2:=sol ve(op(1, si mpl i fy(eval f(eval ([q], res1)[1]))), _Z);
res3:=1/f'(eval f(eval (res2[1], al pha=Pi/2))>0, res2[1], res2[2]);
q:=unappl y(Factor(si mpl i fy(eval f(res3))), al pha);
beta:=arctan((d-s)/m);
del ta:=arccos((u^2+1^2-r^2-(d-s)^2)/(2*u*1));
l1:=eval f(coschi*zeta*R*r/sqrt(R^2-r^2)*sin(beta)/u^2/Pi);
res1:=eval f(Si mpson(Si mpson(l1, r=0..r0, 100), al pha=-theta..theta, 100));
res2:=eval f(Si mpson(Si mpson(l1, r=0..q(al pha), 100), al pha=theta..2*Pi-theta, 100));
Prci dent[k]:=-Re(res1+res2);
l2:=eval f(coschi*zeta*R*r/sqrt(R^2-r^2)/u^2/Pi);
res1:=eval f(Si mpson(Si mpson(l2, r=0..r0, 100), al pha=-theta..theta, 100));
res2:=eval f(Si mpson(Si mpson(l2, r=0..q(al pha), 100), al pha=theta..2*Pi-theta, 100));
Prci tent[k]:=-0.17+2/3*(-0.81)/0.89*Re(res1+res2);
l3:=eval f(sqrt(1-r^2/R^2)*coschi*zeta*R*r/sqrt(R^2-r^2)/u^2/Pi);
res1:=eval f(Si mpson(Si mpson(l3, r=0..r0, 100), al pha=-theta..theta, 100));
res2:=eval f(Si mpson(Si mpson(l3, r=0..q(al pha), 100), al pha=theta..2*Pi-theta, 100));
Pdi fFuse[k]:=-0.83*(1-si gma)^2/3*Re(res1+res2);
uz:=-d-s-2*u*coschi/R*(R-S);
l4:=eval f(-uz/u*coschi*zeta*R*r/sqrt(R^2-r^2)/u^2/Pi);
res1:=eval f(Si mpson(Si mpson(l4, r=0..r0, 100), al pha=-theta..theta, 100));
res2:=eval f(Si mpson(Si mpson(l4, r=0..q(al pha), 100), al pha=theta..2*Pi-theta, 100));
Pspecul ar[k]:=-0.83*si gma*Re(res1+res2);
Ptotal[k]:=-Prci dent[k]+Prci tent[k]+0.83*(Pspecul ar[k]+Pdi fFuse[k]);
end do;
Ptotal[1]-Ptotal[2]+Ptotal[3];
eval (Ptotal[1]+Ptotal[2]+Ptotal[3], si gma=7428);
```

Numerical model

- Divide up spacecraft into simple surfaces, divide surfaces into surface elements
- Starting from surface elements with intrinsic heat, perform ray tracing in all sky directions
- Iteratively calculate specular reflection, diffuse reflection, absorption and re-emission
- May account for heat conduction (simplistic present model distributes absorbed heat evenly along surface)
- When ray departs for infinity, sum spin-axis component
- To calibrate, sum total power of all rays to verify spacecraft thermal power
- Can also be used to estimate torque

Surface elements



Parameter fitting

- Basic concept: in addition to “solving for” orbital elements, also solve for linear factors (q_i) of thermal acceleration. Equation of motion modified with thermal force $F = (1/c)\Sigma q_i P_i$ where the P_i are known.
- Method relies on no *a priori* assumptions about geometry, only about linearity and spin
- Most “correct” method from a modeling/statistical perspective
- Requires orbit estimation and Doppler data in addition to telemetry
- Additional “solve for” parameters may weaken the estimation result
- Linear model is essential; approach not applicable when louvers are partially open

Solving for q

- Analytical values: $q_{\text{elec}} = 0.34$, $q_{\text{RTG}} = 0.009$
- Numerical values: $q_{\text{elec}} = 0.36$, $q_{\text{RTG}} = 0.010$
- “Nominal” initial values: $q_{\text{elec}} = 0.35$, $q_{\text{RTG}} = 0.01$

	Pioneer 10		Pioneer 11	
Initial values	q_E	q_{RTG}	q_E	q_{RTG}
Nominal	0.36	0.012	0.34	0.010
Both 0	-0.26	0.039	-0.04	0.018
One 0	0.52	0.018	0.12	0.006

- Residual acceleration estimate after solving for q using nominal initial values: $a_{\text{P10}} = 4.9 \times 10^{-16} \text{ km/s}^2$, $a_{\text{P11}} = 0 \text{ km/s}^2$
- **Conclusion so far (this is work-in-progress!): “thermal hypothesis” is not contradicted by the 2002 data.**

Thank You!