From stellar systems to the CMB: MOG across 15+ orders of magnitude

Viktor T. Toth

January 18, 2024

Moffat's STVG theory

- Conceptually rooted in Einstein's attempt to explore a nonsymmetric theory for classical unification
- Moffat's take: the antisymmetric part is not Maxwell, it's gravity!
- NGT has stability issues, so why not just postulate a separate nonsymmetric tensor? (MSTG – Metric Skew-Tensor Gravity)
- But wait: a well-behaved nonsymmetric tensor will just be the exterior derivative of a vector field
- A (massive) vector theory with a repulsive force? But that's just Yukawa... Unless the mass and coupling strength themselves are promoted to (scalar) fields
- The result: STVG, Scalar-Tensor-Vector Gravity (Moffat 2006)

MOG is a proper classical field theory

- MOG is a theory of several fields:
  - The tensor field $g_{\mu\nu}$ of metric gravity
  - A scalar field G representing a variable gravitational constant
  - A massive vector field ϕ_μ (NOT a unit timelike field!) responsible for a repulsive force, coupling directly (not through gravity) to matter
  - Additional (possibly nondynamical) scalar degree of freedom determining the mass of ϕ_μ
  - The MOG Lagrangian:

$$\frac{1}{16\pi G}(R + 2\Lambda)\sqrt{-g} - \frac{1}{4\pi}\left[\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{2}\mu^2\phi_\mu\phi^\mu + V_\phi(\phi)\right]\sqrt{-g}$$

$$-\frac{1}{G}\left[\frac{1}{2}g^{\mu\nu}\left(\frac{\nabla_\mu G\nabla_\nu G}{G^2} + \frac{\nabla_\mu\mu\nabla_\nu\mu}{\mu^2}\right) + \frac{V_G(G)}{G^2} + \frac{V_\mu(\mu)}{\mu^2}\right]\sqrt{-g}$$

The MOG acceleration law

- In the weak field, low velocity limit, the acceleration due to a spherically symmetric source of mass M is

$$\ddot{\mathbf{r}} = -\frac{G_N M}{r^3}\mathbf{r}[1 + \alpha - \alpha(1 + \mu r)e^{-\mu r}]$$

An approximate solution for α and μ can be determined from the field equations in the spherically symmetric vacuum case

- For compact sources, the values of α (which determines the coupling strength between matter and ϕ_μ) and μ (which determines the range of ϕ_μ) are determined by the source mass M with formulas fitted using galaxy rotation and cosmology data:

$$\alpha = \frac{M}{\left(\sqrt{M} + E\right)^2}\left(\frac{G_\infty}{G_N} - 1\right), \qquad \mu = \frac{D}{\sqrt{M}},$$

$$D \cong 6250 M_\odot^{1/2}\text{kpc}^{-1}, \qquad E \cong 25000 M_\odot^{1/2}, \qquad G_\infty \cong 20 G_N$$

MOG scales

- The parameters that characterize MOG are **mass scale dependent**
- Contrast this with the MOND formula, which is **acceleration scale dependent**
- The two provide similar results on galactic scales, but diverge on scales much smaller or much larger
- A theory has to work across a wide range of scales, from the solar system to cosmology: More than 15 orders of magnitude

Stellar systems

- For compact sources with mass $\leq \sim 10^7\ M_\odot$, MOG predicts no deviation from GR since $\alpha \sim 0$
- For larger systems, the acceleration law kicks in as $\alpha > 0$. At short range, $\mu r < 1$, the behavior is Newtonian: excess gravity is canceled by the repulsive vector force. When $\mu r > 1$, the behavior is again Newtonian, but with an enhanced gravitational constant, $G = (1 + \alpha) G_N$
- In the intermediate range, $\mu r = 1$, the MOG behavior is consistent with the Tully-Fisher relationship
- Consequently, we expect no deviation from Newtonian (or GR) predictions for stellar systems, wide binaries, or globular clusters

MOG and galaxies

- Unsurprisingly (since that's what inspired its creation) MOG fits spiral galaxy rotation curves quite well
- Other galaxies can be challenging for modified gravity, since they have varied behavior
- Are all those galaxies undisturbed, properly virialized? Which can be trusted to study gravitational dynamics?
- For low mass virialized galaxies, MOG predicts Newtonian behavior
- For higher mass but very compact galaxies, the behavior is again Newtonian except in the outer fringes where $\mu r > 1$

Fundamental parameter-free solutions in Modified Gravity 14

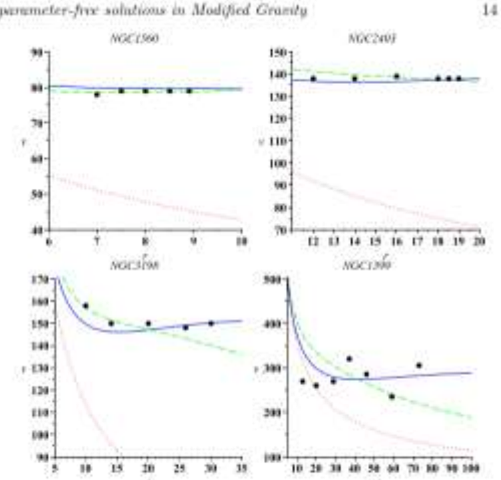


Figure 4. Galaxy rotation curves for a small set of galaxies of varying size. Data points are marked as black dots, current rotational velocity estimates are represented by a solid (blue) curve, while the dashed (green) curve shows velocity estimates in accordance with our earlier work [13, 14]. Mass estimates are as in [13, 14], except for NGC 1399, for which a mass estimate of $M = 5 \times 10^{11}\ M_\odot$ was used. Dotted (red) curve is the Newtonian rotational velocity estimate for those galaxies using the same mass estimates. Radial distances are measured in kpc, masses in $M_\odot$.

Scalar–tensor–vector–gravity and NGC-1277 3

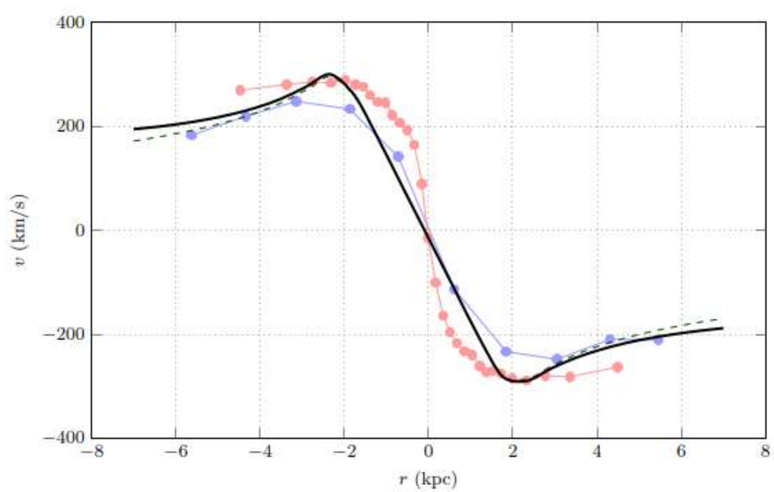


Figure 1. Comparison of the rotation curve predicted by MOG (solid black line) or Newtonian gravity (dashed green line), using a very simple mass model of $M = 1.3 \times 10^{11}$ solar masses concentrated within the inner 2 kpc, against the data presented in (Comerón et al. 2023) (blue dots) as well as (Ferré-Mateu et al. 2017) (red dots).

MOG and galaxy clusters

- The “Bullet Cluster” 1E0657-558 was heralded as the modified gravity “killer”, yet MOG has no trouble fitting the data
- Other clusters, like the “train wreck” cluster Abell 520, that challenge the CDM paradigm, also work well

42 *J. R. Brownstein and J. W. Moffat*

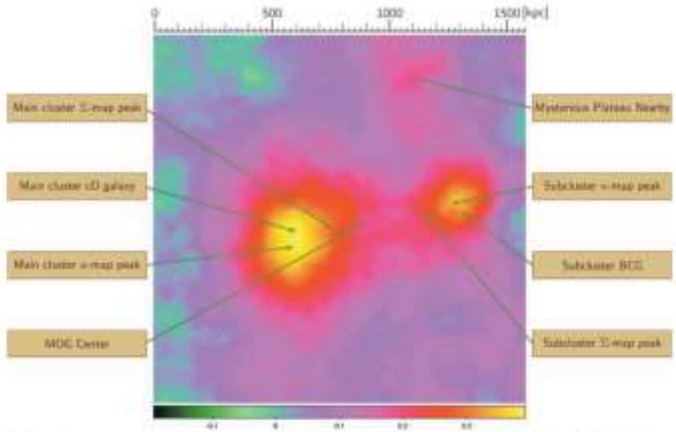


Figure 12. The surface density κ-map reconstructed from strong and weak gravitational lensing. Main cluster and subcluster of the Bullet Cluster 1E0657-558, 2006 November 15 data release κ-map (Clowe et al. 2006c) observed peaks (local maxima) and Σ-map observed peaks are shown for comparison. The CD galaxy of the main cluster, the BCG of the subcluster and the MOG predicted gravitational centre are shown. J2000 and map (x, y) coordinates are listed in Table 1.

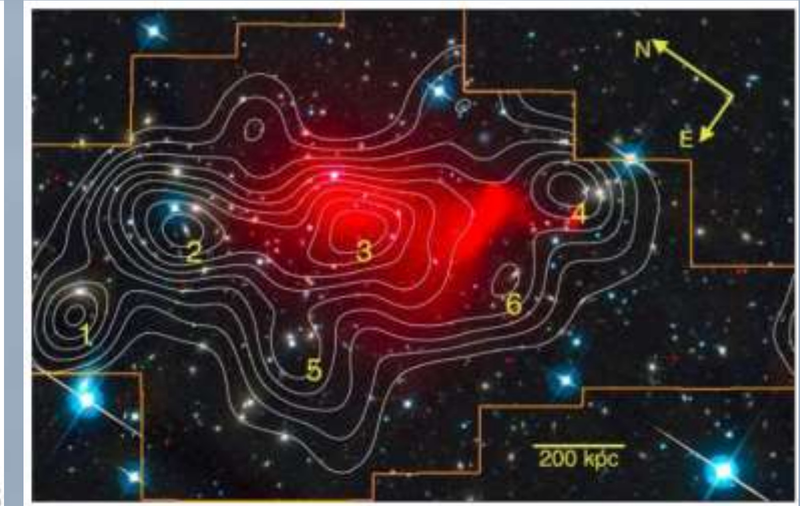


Figure 2. The Abell 520 cluster with distribution of galaxies, gas and contours from weak lensing [3].

MOG and cosmology

- We tested MOG extensively against cosmological results
- MOG predicts the same CMB angular power spectrum as the standard model
- Another important test is the matter power spectrum. A “pure matter” theory predicts unit baryonic oscillations and a curve with the wrong slope
- MOG’s variable gravitational constant restores the slope, matching observation
- The unit oscillations are present, but are wiped out by statistics in the analysis; to the extent that they can be seen, they are consistent with MOG

Figure 2. The effect of window functions on the power spectrum is demonstrated by applying the SDSS luminous red galaxy survey window functions to the MOG prediction. Baryonic oscillations are greatly dampened in the resulting curve (solid red line). A normalized linear ΛCDM estimate is also shown (thin blue line) for comparison.

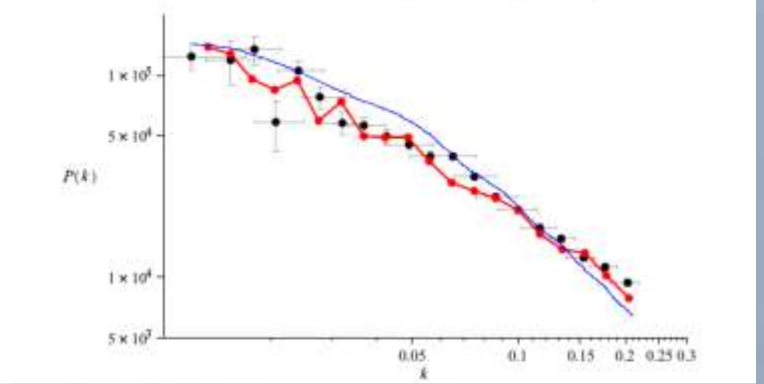
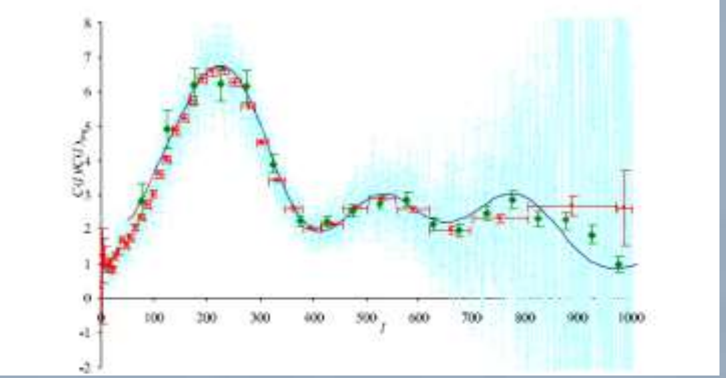


Figure 3. MOG and the acoustic power spectrum. Calculated using $\Omega_M = 0.3$, $\Omega_b = 0.035$, $H_0 = 71$ km/s/Mpc. Also shown are the raw WMAP 3-year data set (light blue), binned averages with horizontal and vertical error bars provided by the WMAP project (red), and data from the Boomerang experiment (green).



MOG challenges

- MOG results are based, in part, on a separately postulated test particle Lagrangian. The theory still needs integration, properly accounting for the coupling constant between matter and MOG fields
- In the solar system, MOG must meet stringent post-Newtonian observational results. It remains unclear to me how a possible nonzero galactic background scalar field might affect, e.g., gravitational light deflection
- The mass dependence derivation of α and μ is heuristic. A more robust derivation is lacking
- MOG does not run into the same trouble as bimetric theories when contrasted with multimessenger observations, but a robust formulation of MOG gravitational radiation remains to be found.

Thank you!

- Any questions?